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APPLIED MATHEMATICS

THE PERFORMANCE OF A FULLY SUBMERGED PROPELLER
IN REGULAR WAVES

bу

J.H. Mccarthy, W.H. Norley and G.L. Ober



HYDROMECHANICS LABORATORY
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THE PERFORMANCE OF A FULLY SUBMERGED PROPELLER IN REGULAR WAVES

 $\mathbf{B}\mathbf{y}$

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NOMENCLATURE

- a Wave Height at Surface = 2r_o
- $b_{O,7}$ Blade Section Length at 0.7 Radius
- Distance between Surface Orbit Centers and Still Water Level = $\frac{\pi \pi_o^2}{\lambda}$
- D Propeller Diameter
- e Propeller Efficiency = $\frac{T V_a}{2\pi Q m} = \frac{K_{\pm}}{K_q} \cdot \frac{J}{2\pi}$
- f_e Frequency of Encounter of a Propeller and a Wave Traveling in the Same or Opposite Direction = $\frac{\bigvee \pm \bigvee_{\omega}}{\lambda}$
- h Distance from center of Surface Orbit Circle to that of the Orbit Circle Belonging to any given Subsurface
- h Distance from Still Water Level to any Point Below the Surface
- J Speed Coefficient Based on Carriage Speed = $\frac{\sqrt{}}{mD}$
- J_a Advance Coefficient in Waves Corrected for Mean Orbital Velocity = $\frac{V_a}{mD}$
- K_t Thrust Coefficient = $\frac{T}{\varrho m^2 D^4}$
- K_q Torque Coefficient = $\frac{Q}{\rho \, m^2 \, D^5}$
- n Instantaneous Revolutions per Unit of Time
- P Propeller Pitch
- p Distance of Propeller Shaft Centerline Below Still Water Level
- Q Torque
- R Propeller Radius
- R_e Reynolds Number = $b_{0.7} \frac{\sqrt{V_a^2 + (0.7 \pi m D)^2}}{V}$
- r Orbit Radius of Surface Trochoidal Wave
- r Local Orbit Radius of Trochoidal Wave

Subscript c Crest

Subscript m Average of Crest and Trough

Subscript t Trough

- T Thrust
- v Local Orbital Velocity at a Point in the Propeller Disc
- Mean Orbital Velocity over the Propeller Disc
- V Speed of Towing Carriage
- V_a Speed of Advance = $\vee \pm \overline{\kappa}$
- V_{W} Wave Celerity = $\sqrt{\frac{\lambda_{Q}}{2\pi}}$
- λ Wave Length
- ν Kinematic Viscosity
- P Density
- ρ,φ Polar Coordinates of a Point in the Propeller Disc

ABSTRACT

The performance of a 16-inch, 6-bladed propeller in waves was determined experimentally, and the records of the time-dependent fluctuating propeller forces are analyzed in this report. Various wave lengths, wave heights, and frequencies of encounter were investigated with the propeller advancing normal to the wave crests at each of two fixed shaft centerline submergences.

The results indicate that the orbital velocities of the waves have a significant effect, causing fluctuations of the advance coefficient and corresponding fluctuations of the thrust and torque coefficients. The local advance coefficients have been determined by calculating the mean orbital velocities of the waves in way of the propeller disc from trochoidal wave theory. The thrust and torque coefficients for the propeller when operating below wave crests and wave troughs have been plotted against their local advance coefficients, and the results are found to be in good agreement with the open-water characteristic curves for the propeller in uniform flow. It is concluded that the performance of a propeller in waves may be predicted from a quasi-steady state analysis.

In addition, a general expression for a wave's mean orbital velocity in way of a propeller disc, for various depths below the crest and trough, is determined for a range of waves normally encountered at sea. Non-dimensional plots show mean orbital velocities in a wave crest and wave trough for ratios of propeller centerline submergence to wave length, and propeller diameter to wave length.

INTRODUCTION

Seaworthiness and ship performance in waves have, over the years, received considerable research effort. 1,2,3,4 Because of the interest in the overall seagoing qualities of a ship, attention was directed to the propeller performance in waves. A project was initiated in February 1957 to evaluate the time dependent propeller forces while operating in unsteady flow as obtained by tests in waves.* It was desired to acquire fundamental information on propeller action in waves for the improvement of propeller design. The wave making facility and equipment available at the David Taylor Model Basin were suitable for the project. In the tests undertaken, measurements were made on a time basis of the propeller fluctuating thrust, torque and rpm and the location of the propeller in the wave. The forward speed of the propeller was held constant for each run.

EQUIPMENT AND TEST PROCEDURE

A 16-inch six-bladed bronze propeller (Propeller 3643) was tested in waves and in uniform flow. Figure 1 shows the drawing and dimensions of the propeller.

For the tests in waves, the TMB 35-horsepower dynamometer was used for measuring the instantaneous torque, thrust, and rpm. A wave height probe was used to measure the instantaneous height of waves in the plane of the propeller. This equipment together with Sanborn recording instruments were installed on Carriage II of the deep basin which has a wave making facility. Instantaneous total wave height and total rpm, together with the fluctuating components of thrust and torque were recorded

^{*} A preliminary report on the findings of the project was presented by W.H. Norley in a paper, "Propeller Performance in Unsteady Flow," given at the American Towing Tank Conference held at Berkeley, California on 31 Aug 1959

on the Sanborn Recorder. The steady-state portion of thrust was recorded separately thus increasing the sensitivity for measuring the fluctuating value.

The tests in waves were run with the shaft centerline submerged 18 inches and 21 inches below the still water surface level. Wave lengths for both depths were 40 feet, 30 feet, and 20 feet. The nominal wave lengths, wave heights, and carriage speeds were as follows:

Wave Length (λ)	20'		30'		40'
Wave Height (a)	6"	6"	9"	12"	12"
a/\(\chi\)	1/40	1/60	1/40	1/30	1/40
Carriage Speed (V)	8 ft/sec	8 ft/sec	6 ft/sec 8 ft/sec 10 ft/sec	8 ft/sec	8 ft/sec

The minimum immersion to the propeller tip is 0.27D and occurs in way of the wave trough for λ = 30 ft, a = 12 ins., p = 18 ins.

A train of waves was set up in the basin in a direction opposite to the direction of motion of the carriage. A sketch of the propeller test arrangement is given in Figure 2. In the region of the basin where the waves were the most uniform, transient records were obtained for wave height, thrust, torque, and RPM. All recording pens were lined up together on the same time base so values could be easily obtained from the records for any phase of a specific wave.

The tests in uniform flow (i.e., without waves) were run using the 35-horsepower dynamometer, at a constant rotational speed of 15.2 rps at various carriage speeds, with the propeller shaft centerline submerged one and a half feet and four feet. Three separate uniform flow tests were run for the propeller on different dates. Because of scatter between the performance results for the propeller from each test, the curves were faired with most weight placed on the 8 April 1958 test, which was run at the time of the wave tests.

RESULTS

The results of the propeller characterization in uniform flow are presented in Figure 3. A sample of the oscillograph records of performance of the propeller in waves is shown in Figure 4. An example of propeller performance given in terms of the usual nondimensional coefficients is shown in Figure 5, where the coefficients have been evaluated from the original continuous records at 0.2 second intervals.

As indicated above the propeller and waves were traveling in opposite directions of motion. An examination of the records indicated that the maximum thrust and torque occurred while the propeller was below the troughs of the waves, and the minimum thrust and torque occurred while below the crests of the waves. In most cases, the maximum rpm was obtained below the crest and the minimum rpm below the trough. The frequency of fluctuation ($\frac{1}{100} = \frac{1}{100} \frac{1}{100} = \frac{1}{100} =$

In Figure 6 all the test results for fluctuating thrust, torque and RPM are plotted against average speed coefficient $J_m \left(J_m = \frac{V}{N_m D} \right)$. The fluctuating thrust, torque and RPM are plotted as percentage changes relative to the average values. It is seen that the percentage changes in thrust and torque tend to increase exponentially with increasing average speed coefficient, and the precentage change in RPM tends to increase linearly with increasing average speed coefficient. It should be observed that in Figure 6 the results for both the 18 inch and 21 inch propeller submergences have been plotted together. Ideally there should be a difference in the values of flucuating thrust, torque, and RPM for different propeller submergences. Figure 6, however, indicates that the difference in propeller submergence was not appreciable enough to significantly change the fluctuating quantities measured.

In Figures 7, 8 and 9 faired curves for the test results shown in Figure 6 are plotted against average speed coefficient. The faired results shown indicate that the percentage changes in the propeller st thrust and torque increase with:

- (1) Increasing average speed coefficient (J_m)
- (2) Increasing wave height-propeller diameter ratio $(\frac{a}{D})$
- (3) Decreasing propeller speed-wave celerity ratio $(\frac{V}{V_w})$
- (4) Increasing wave height-wave length ratio $(\frac{a}{\lambda})$

These results are interdependent and are in agreement with existing propeller theory and tropchoidal wave theory. One further characteristic of a propeller in waves which is not indicated from the test results is that the percentage changes in thrust and torque decrease with increasing submergence of the propeller. As mentioned above the range of submergence depths at which the propeller was tested was not great enough to show this.

Nondimensional thrust and torque coefficients were evaluated for the crest and trough positions and were plotted over the range of speed coefficients tested, where speed coefficient ($J = \frac{\vee}{mD}$) was based on carriage speed and instantaneous RPM. The average of several instantaneous values was used in determining these coefficients. An example of such performance curves is shown in Figure 10. The results shown in Figure 10 are for a 40-foot wave with a height of 12 inches and a wave velocity of 8 fps. The propeller shaft centerline was submerged 18 inches below the still water surface level. This is a typical example of the results obtained for every wave condition.

Additional analysis of the test results showed that it was possible to correlate the thrust and torque coefficients in the crests and troughs by plotting them against advance coefficient, ($\mathcal{J}_{\alpha} = \frac{\vee \pm \sqrt{\Gamma}}{M}$) i.e.; advance coefficient accounts for the effect of the orbital velocities of the waves.* Figure 11 shows the results given in the preceding figure when thrust and

^{*} Details of computing the mean orbital velocity are given in the Appendix.

and torque coefficient are plotted against advance coefficient. In addition, the thrust and torque coefficients for uniform flow have been included for comparison.

The results of all tests were then similarly plotted and are shown in Figure 12 for the propeller in the below crest and trough position for all waves conditions. (The advance coefficients are corrected for mean orbital velocity of the waves). As before, the uniform flow performance curves are included.

In Figure 15 and Figure 16 the mean orbital velocities in way of the the propeller disc are plotted for the propeller operating below crests and troughs. The plots are nondimensionalized and are based on a derivation given in the Appendix.

DISCUSSION

In calculating the advance coefficient for a propeller below the crests and troughs of waves, the mean orbital velocity has been taken to be the "mean" over the entire propeller disc (See Appendix). The true "mean", however, if calculated for an <u>instantaneous</u> position of the propeller blades, should take into account the orbital velocities only in way of the propeller blades, and not over the entire propeller disc. Two approximations are introduced by calculating the "mean" over the entire propeller disc:

- 1. The effect of hub radius is not included.
- 2. The effect of the number of blades is not included.

The first approximation is justified if the hub radius is small in comparison to the tip radius. For the propeller tested the hub radius is 20 percent of the tip radius, and the cross sectional area of the hub represents only 4 percent of the total disc area. It is evident then that the first approximation introduces a negligible error into the calculated mean orbital velocity.

The second approximation for calculating <u>instantanteous</u> advance coefficient can be justified only when a propeller has a large number of blades since only then would the blades approach the condition of working over the entire propeller disc at any given instant below a crest or trough of a

wave. The second approximation, however, is valid when calculating the mean advance coefficient, regardless of the number of blades, for all possible blade positions of a propeller when working below a crest or trough. For propellers with a large number of blades the mean and instantaneous advance coefficients will be nearly identical but it is noted that for a small number of blades the instantaneous values could be greater or less than the mean value. Further, in order for a mean advance coefficient in a wave crest or trough to exist, the time for one complete revolution of the propeller must be equal to or less than the length of time that the propeller is operating below a wave's crest or trough. In this case the propeller blades would sweep over the entire disc area when below a crest or trough and the mean orbital velocity to the propeller blades would then be the "mean" over the entire disc area. For trochoidal waves the crests and troughs are relatively flat, and it can be assumed that a propeller will pass below them for a finite length of time.

Since no record of the instantaneous position of the blades was made for each instantaneous thrust and torque measured it was not possible to calculate the corresponding mean orbital velocity at each blade. Instead, average values of the thrust and torque measured at the crests and troughs for each test condition were adopted and mean orbital velocities over the entire disc were assumed to be valid in calculating an advance coefficient to correspond with each mean thrust and torque. In all tests run for this report the time for one complete propeller revolution was less than the length of time that the propeller operated below a crest or trough*. Hence, the calculation of a mean orbital velocity over the entire propeller disc does justifiably correspond to a mean propeller thrust or torque, and the second approximation is valid in analysing the test results.

It is interesting to note from Figure 4 (a typical example of data taken) that the instantaneous thrusts and torques measured below crests and troughs do not vary significantly, due to the large number of blades (6) for the propeller tested. For a three bladed propeller the variations would be more appreciable.

^{*} For example see Figure 5. The time duration of a crest or trough is approximately 0.2 seconds. For the propeller operating at 8 rps, 1/8 < 0.2.

It is evident from Figures 4 through 10 that waves can have a significant effect on the thrust and torque developed by a propeller, causing fluctuations of the advance coefficient and corresponding fluctuations of the thrust and torque. As can be seen from Figures 11 and 12, the thrust and torque coefficients for the propeller in waves agree well with the uniform flow (steady-state) results, when plotted against advance coefficient ($J_{\alpha} = \frac{\bigvee \pm \sqrt{N}}{N}$), where advance coefficient takes into account the orbital velocities of the waves in way of the propeller disc. The greater scatter at high J_{α} values is probably due to reduced sensitivity in the dynamometer readings taken at low thrust and torque values.

The results indicate that, within the range of the test conditions, thrust and torque developed by a propeller operating below a wave crest or trough can be estimated from the uniform flow propeller performance curves. This means that a quasi-steady state calculation of fluctuating thrust and torque for a propeller in waves will give good predictions, and that unsteady effects may be neglected. It should be noted that the frequencies of encounter of the propeller and waves in the tests undertaken were low (0.5 to 1.0 cps) and that only within a low range of frequencies of encounter ($f_e = \frac{V \pm V_W}{\lambda}$) do these tests justify neglecting unsteady effects in calculating thrust and torque fluctuations. It is further noted, however, that in most practical examples the frequency of encounter will fall below 1.0 cps.

It is evident from Figure 12, that even at high slip values (low J_a) there is no breakdown in propeller thrust or torque, indicating that propeller immersion was sufficient enough to avoid cavitation and air suction effects discussed in References 1 and 7. The tests reported here, then are for the case of the truly fully-submerged propeller.

While it is true that the experimental results were obtained only for the case of a propeller traveling opposite to the direction of travel of a wave, it is logical to assume that the general conclusions arrived at based on these results apply equally to the case of a propeller traveling in the same direction as the direction of travel of a wave. However,

in considering each case proper attention must be paid to the relative direction of the propeller and the direction of the mean orbital velocity of the wave at its crest and trough.

THE PRACTICAL PROBLEM

Let us consider the following practical problem for the prototype of the propeller tested for this report, where the wave characteristics are:

a... 20 feet
$$\frac{a}{\lambda} = .050$$
... 400 feet
$$\frac{p}{\lambda} = .050$$
p... 20 feet

For the prototype propeller assume:

D = 10 feet . . .
$$\frac{D}{\lambda}$$
 = .025
RPM = 180
V = 21.70 fps
J = .723 (design advance coefficient)

From Figure 15 and Figure 16 or from Equation (18) of the Appendix we may determine the mean orbital velocity $(\bar{\mathbf{v}})$ of the wave crest or trough in way of the propeller:

Crest:
$$\frac{\overline{N_c}}{\sqrt{\lambda q}} = .041$$
 $\overline{v_c} = 4.65 \text{ fps}$
Trough: $\overline{v_t} = .051$ $\overline{v_t} = 5.79 \text{ fps}$

The advance coefficient $(J_a = \frac{V \pm N}{MD})$ may then be calculated for the crest and trough positions of the propeller, when the propeller is advancing normal to the wave crests. Referring to Figure 13 it is evident that when the propeller is traveling in the same direction as the direction of wave travel:

Crest:
$$J_a = \frac{V - \overline{N}_c}{M D}$$
Trough: $J_a = \frac{V + \overline{N}_{\frac{1}{N}}}{M D}$

And when traveling in opposite directions:

Crest:
$$J_a = \frac{V + \overline{N}_c}{MD}$$

Trough: $J_a = \frac{V - \overline{N}_t}{MD}$

The following tables present the values for the advance coefficient in this example, together with K_T , K_Q , and propeller efficiency (e) as obtained from the characteristic curves for the propeller in uniform flow (Figure 3). The percentage thrust, torque and efficiency fluctuations above and below the still water values are also tabulated

Table 1
Propeller Traveling in Same Direction as Wave

		J_a	K_{t}	К _q	е	<u> </u>	AKB	De
Still Water:	.723	.723	.210	.0356	.680	^ ★	K &	
Crest:	.723	.568	.285	.0456	.565	+36%	+28%	-17%
Trough:	•723	.916	.109	.0217	•735	-48%	- 39%	+8%

Table 2
Propeller Traveling Counter to Direction of Travel of Wave

	J	J_a	K_{t}	$\kappa_{\mathtt{q}}$	е	<u>ΔK*</u> K+	<u>AK</u> F	<u>se</u>
Still Water:	.723	.723	.210	.0356	.680	^*	rd	_
Crest:	.723	.878	.130	.0246	.743	-38%	-31%	+9%
Trough:	.723	.531	.303	.0480	•533	+44%	+35%	-22%

In the above example RPM is assumed to remain constant. The test results given earlier indicated that RPM fluctuations of up to about ± 3% occurred for the model propeller in waves. It is believed, however, that for a fully-submerged prototype propeller, where the mass moments of inertia are large for the propeller, shafting, and machinery, the RPM fluctuations would be reduced to such an extent that changes in RPM may be neglected

in estimating fluctuating thrust and torque.

In the tabulated results given above it is interesting to note that the percentage changes in thrust, torque, and efficiency relative to the still water values, vary in magnitude and sign according to the relative directions of travel of the propeller and wave. The maximum changes in thrust and torque, relative to the still water values, occur in the wave trough for the case of the propeller traveling in the same direction as the wave.

The example worked above assumes the following:

- 1. The propeller runs at constant submergence below the still water surface.
- 2. The plane of the propeller disc remains normal to the still water surface.
 - 3. The wave which passes the propeller is trochoidal.
 - 4. The propeller remains fully-submerged.

None of these conditions is necessarily fulfilled for a propeller operating behind a ship in a head or a following sea. A ship in a head or following sea will pitch and heave and the submergence depth and inclination of the propeller will be continuously changing. Further, the ship, in close proximity to the propeller, may alter the trochoidal wave. Ship wake may also have a significant effect. In addition to these considerations for the case of a propeller behind a ship, it must also be remembered that in actual cases the propeller may not always be fully-submerged, and that air suction by and cavitation of the propeller may occur. For measurements in waves of changes in actual ship propeller torque and RPM in the behind condition the reader is referred to Reference 8.

CONCLUSIONS

- 1. Waves can have a significant effect on the instantaneous thrust and torque developed by a propeller, causing fluctuations of the advance coefficient and corresponding fluctuations of the thrust and torque.
- 2. For wave crest and trough, a correlation of experimentally measured open water propeller thrust and torque coefficients with advance coefficient, taking proper account of the mean orbital velocities of the waves, is in good agreement with the uniform flow characteristic performance curves for the propeller.
- 3. For the low frequencies of encounter of a propeller and waves, unsteady effects may be neglected in calculating the instantaneous thrust and torque of a propeller in a wave crest or trough. A quasi-steady state calculation, based on the uniform flow characteristic performance curves for a given propeller, may be made to estimate the propeller's instantaneous thrust and torque when operating fully-submerged in a wave crest or trough.
- 4. In the case of a propeller operating behind a ship in waves, further work should be done to determine the effect of ship motions on fluctuating propeller forces, the effect of the ship on the ideal trochoidal wave assumed to exist at the propeller disc, and the effects of propeller, shafting and driving machinery on RPM fluctuations.

ACKNOWLEDGEMENTS

Credit for the development and application of the instrumentation and for the testing is given to Goodwin Ober and James Peck. Jakob Auslaender formulated the equations and programmed the calculation of the mean orbital velocities over the propeller disc area. Other members of the Propeller Branch contributed help to the experimentation and data reduction.

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APPENDIX

DETERMINATION OF A WAVE'S MEAN ORBITAL VELOCITY IN WAY OF A PROPELLER

In an open water test of a propeller in still water the speed of advance is simply the carriage speed. When, however, the open water tests are run in waves the speed of advance of the propeller is modified by the orbital velocities of the waves. Therefore, in order to determine the mean speed of advance of a propeller in waves at any given instant the mean orbital velocity of the wave in way of the propeller disc must be determined.

In all tests carried out for this report the direction of motion of the carriage towing the propeller was counter to the direction of travel of the waves. (See Figure 13). Based on trochoidal wave theory, it is evident from Figure 13 that in way of a wave crest the velocity of advance of the propeller will be greater than the carriage speed, and that in way of a wave trough the velocity of advance of the propeller will be less than the carriage speed. If $\bar{\mathbf{v}}$ is the wave's mean orbital velocity integrated over the propeller disc and V the carriage speed it follows that the velocity of advance of the propeller (Va) will be given by:

$$V_a = V \pm \bar{N} \tag{1}$$

(where $\bar{\mathbf{v}}$ is positive at crest and negative at a trough for a propeller traveling counter to the direction of wave travel.)

In the following derivation the wave mean orbital velocity will be taken as the mean volume orbital velocity of the wave integrated over the propeller disc. Hence, $\bar{N} = \int_{-\pi}^{R} \int_{-\pi}^{2\pi} N \rho \, d\phi \, d\rho$

$$\bar{x} = \int \int x \rho d\phi d\rho \qquad (2)$$

where v is the local wave orbital velocity at a point in the propeller disc, and the other symbols are as defined in Figure 14.

From trochoidal wave theory 9

$$N\Gamma = \sqrt{\frac{2\pi a}{\lambda}} \cdot \pi \tag{3}$$

where, r = local orbital radius

 λ = wave length

Substitution of (3) into (2) yields:*

$$\frac{\overline{N}}{\sqrt{\lambda q}} = \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \frac{1}{\lambda R^2} \int_{0}^{R} \int_{0}^{2\pi} \pi \, d\phi \, \rho \, d\rho \tag{4}$$

In order to integrate equation (4) r must be found in terms of ρ and ϕ . From Reference 9:

$$\pi = \pi_0 e^{-\frac{2\pi h}{\lambda}}$$
(5)

where, r_0 = orbit radius of surface trochoidal wave

h # depth from center of surface orbit circle to that of the orbit circle belonging to any given subsurface.

If subscripts c, t, denote crest and trough respectively, from Figure 9:

$$h_c = h_0 + d + r_c$$
 $h_t = h_0 + d - r_t$
(6)

If we let $A = \pi_0 e^{-\frac{2\pi}{\lambda}(\lambda_0 + d)}$, from equation (5): $\pi_c = A e^{-\frac{2\pi}{\lambda}\pi_c}$

$$\pi_{c} = A e^{-\frac{2\pi}{\lambda}} \pi_{c}$$

$$\pi_{+} = A e^{\frac{2\pi}{\lambda}} \pi_{+}$$
(7)

Let
$$\ll_c = -\frac{2\pi}{\lambda}$$

$$\alpha_{\star} = \frac{2\pi}{\lambda}$$

^{*} Strictly speaking \bar{v} should be determined by integration from hub to tip of the propeller. The above simplification is justified when the hub radius is small compared to the tip radius.

Then,

$$\pi_{c} = A e^{\alpha_{c} \pi_{c}}$$

$$\pi_{+} = A e^{\alpha_{+} \pi_{+}}$$
(5)

Equations (8) are identical except for subscript; hence, temporarily drop the subscript and let

$$X = A \propto , \xi = \frac{\pi}{A}$$

and obtain:

$$\xi = e^{X\xi} \tag{9}$$

Equation (9) can be solved for ξ and yields a series solution

$$\xi = 1 + X + \frac{3X^{2}}{2!} + \frac{4^{2}X^{3}}{3!} + \dots$$

$$\xi = \sum_{k=0}^{\infty} \frac{(k+1)^{k-1}}{k!} \cdot X^{k} \quad (X < \frac{1}{e}) \quad (10)$$

The maximum value of X occurs when $e^{-2\pi(A_0+d)}$ is a maximum, or when (h_0+d) approaches a minimum. The minimum value for (h_0+d) for a propeller which is always submerged occurs when $R \to 0$ and $r_0 \to 0$, noting that $d = \frac{\pi r_0^2}{\lambda}$ (see Figure 13). Hence:

$$\lim_{R\to0}e^{-2\pi(h_0+d)}\longrightarrow 1$$

Therefore:

$$\lim_{\tau_{\bullet} \to 0} X \longrightarrow 2\pi \frac{\tau_{\bullet}}{\lambda}$$

$$R \to 0$$

but, $\frac{2\pi \Lambda_{\bullet}}{\lambda}$ (e for convergence

or, $\frac{\pi}{\lambda}$ <.0565 for convergence

This condition is met for all waves in this report, and is satisfied for the highest waves, where $\frac{\hbar_0}{\lambda}$ = .025 is generally assumed to be the maximum value possible.

From equation (10), Substitute $\xi = \mathbb{L}$ and obtain

$$T = A \sum_{k=0}^{\infty} \frac{(k+1)^{k-1}}{k!} \cdot X^{k}$$
(11)

From Figure 10,

Hence,

$$X = A \propto = \left(\pm \frac{2\pi}{\lambda}\right) r_o e^{-\frac{2\pi}{\lambda}(p - \rho \cos \phi + d)}$$
 (12)

where - denotes crest and + denotes trough

Substituting (12) into (11):

$$\pi = \sum_{k=0}^{\infty} \frac{(k+1)^{k-1}}{k!} \left[\pi_{\bullet} e^{-\frac{2\pi}{\lambda} (p - p \cos \phi + d)} \right]^{k+1} (\pm \frac{2\pi}{\lambda})^{k}$$
 (13)

From Reference 9 we obtain:

$$d = \frac{\pi r_0^2}{\lambda}$$

Letting,

$$\frac{1}{h}(k) = \frac{2\pi}{\lambda} (k+1)$$

$$b(k) = \left[\frac{(k+1)}{k!}^{k-1} \right] \left[\pi_o e^{-\frac{2\pi}{\lambda} (-p + \frac{\pi \pi_o^2}{\lambda})} \right]^{k+1} (\pm \frac{2\pi}{\lambda})^k$$

we obtain:

$$\pi = \sum_{k=0}^{\infty} b(k) e^{\frac{1}{2}(k)} \rho \cos \phi \qquad (14)$$

Substituting (14) into (4):
$$\frac{\overline{N}}{\sqrt{\lambda q}} = \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \frac{1}{\lambda R^2} \int_{0}^{R} \left[\sum_{k=0}^{2\pi} b(k) e^{\frac{1}{2}(k)} \rho \cos \phi \right] d\phi \rho d\rho \quad (15)$$

Let
$$\phi = Y + \pi$$
, $\cos \phi = -\cos Y$, and
$$\frac{\overline{N}}{\sqrt{\lambda_q}} = \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \frac{1}{\lambda_{R^2}} \int_{0}^{\infty} \int_{0}^{\infty} b(k) e^{-\frac{1}{2}(k)} e^{\cos Y} dx dx dx$$

The solutions for the integrals within the brackets are identical, and given in Reference 11, p 145, equation (156):

Hence:

$$\frac{\overline{U}}{\sqrt{\lambda q}} = \frac{2\sqrt{2\pi}}{\lambda R^2} \int_{k=0}^{R} b(k) I_o[k(k)\rho] \rho d\rho \qquad (16)$$

where,

$$I_o\left[\frac{1}{2}(k)\rho\right] = \sum_{m=0}^{\infty} \frac{\left[\frac{1}{2}(k)\rho\right]^{2m}}{(m!)^2}$$
and $\frac{1}{2}(k)$ and let

Substitute b(k) and $\frac{1}{k}(k)$ and let

$$E = \pi \frac{R}{\lambda}$$

$$B = \frac{2\pi \pi_o}{\lambda} e^{-2\pi (\pi \frac{\pi_o^2}{\lambda^2} + \frac{\pi_o}{\lambda})}$$

and obtain:

$$\frac{\overline{N}}{\sqrt{\lambda_a}} = \frac{B}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(k+1)^{k-1}}{k!} \left[\pm B \right]^k \sum_{m=0}^{\infty} \frac{\left[E(k+1) \right]^{2m}}{m! (m+1)!}$$
where - B is at crest and + B is at trough:

Equation (17) was programmed on an IBM 704 computer and the results are shown in Figure 15 and Figure 16 for the non-dimensionalized mean orbital velocities at crests and trough of trochoidal waves. The results are plotted for $\frac{\alpha}{\lambda} = 0$ to $\frac{\alpha}{\lambda} = .05$, $\frac{1}{\lambda} = .025$ to $\frac{\alpha}{\lambda} = .000$ and $\frac{\Omega}{\lambda} = .000$ to $\frac{\Omega}{\lambda} = .25$.

The series solution for the mean orbital velocity given in equation (17) converges very rapidly. In expanding the series solution, terms of the type B^X E^Y will appear. If all B^X E^Y terms, where $\times + \checkmark$ > 5 are neglected, the following approximate solution is obtained:

$$\frac{\overline{N}}{\sqrt{\lambda}q} = \frac{1}{\sqrt{2\pi}} \left[(B \pm B^2 + \frac{3}{2} B^3 \pm \frac{B}{3} B^4 + \frac{125}{24} B^5) + E^2 (\frac{B}{2} \pm 2B^2 + \frac{27}{4} B^3) + E^4 (\frac{B}{12}) \right]$$
(18)

where the - is at crest and the + is at trough.

For the range of parameters given in Figure 15 and Figure 16, the approximate solution (18) is in excellent agreement with the exact solution (17).

From Figure 15 and Figure 16, it may be noted that as $\frac{Q}{\lambda} \rightarrow 0$ the mean orbital velocity rapidly approaches the value of the local orbital velocity at the centerline of the propeller. For $\frac{D}{\lambda} < .05$ the mean orbital velocity is approximately equal to the local orbital velocity at the propeller centerline.

PROPELLER 3643

NUMBER OF BLADES	6
EXP.AREA RATIO	0.992
MWR	0.324
BTF (AT 0.3R)	0.043
P/D (AT 0.7R)	1.033
DIAMETER	16.000 ins
PITCH (AT 0.7R)	16.528 ins
ROTATION	R. H.
TEST n	15.2 rps
TEST V _a	6.1 - 22.9 fps
-	

TESTED FOR..... BUSHIPS
DESIGNED BY..... TMB
TMB DRAWING P-3643

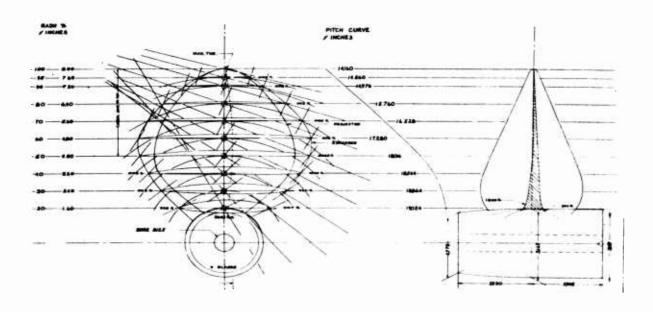
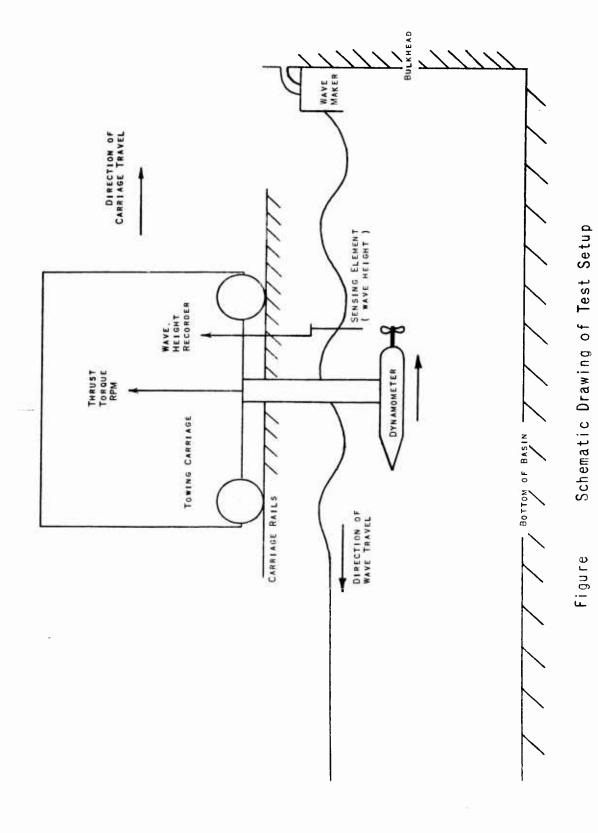


Figure 1 Drawing and Propeller Dimensions



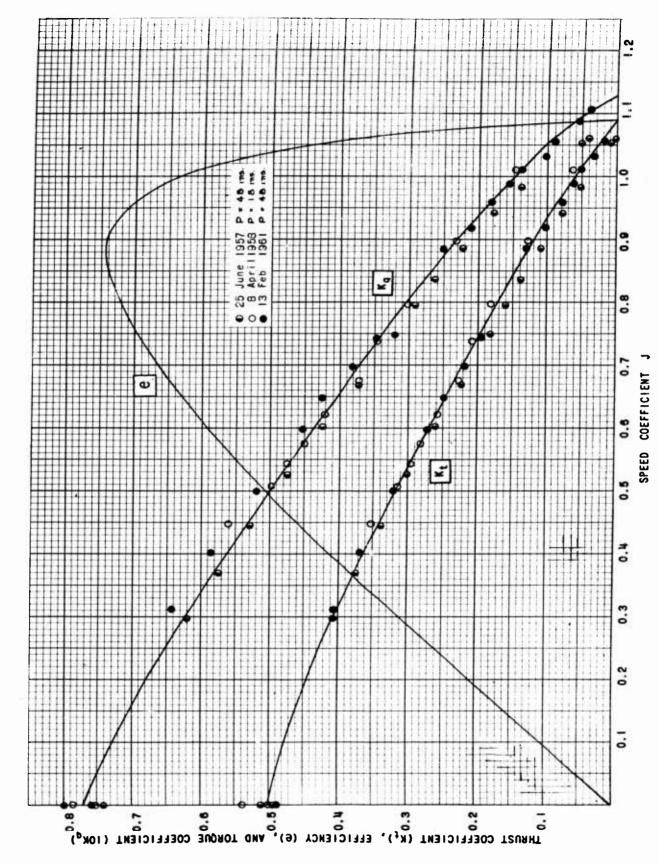


Figure 3 - Propeller Characteristic Curves for Uniform Flow

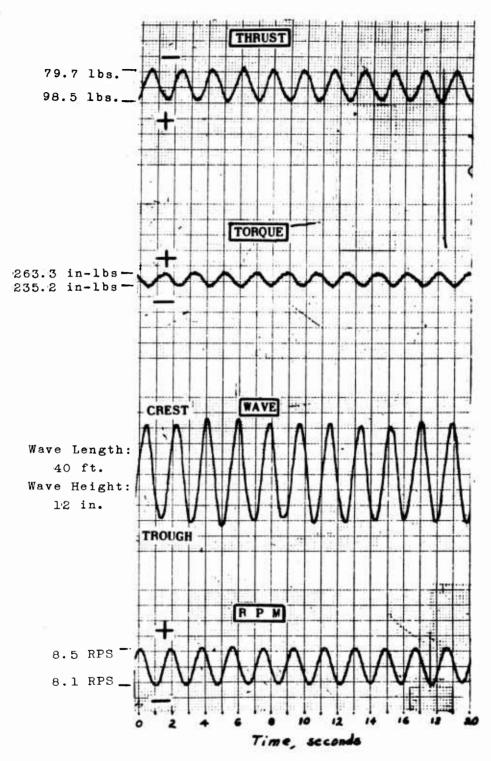


Figure 4 A. Sample of Oscillograph Records of Propeller Performance in Waves

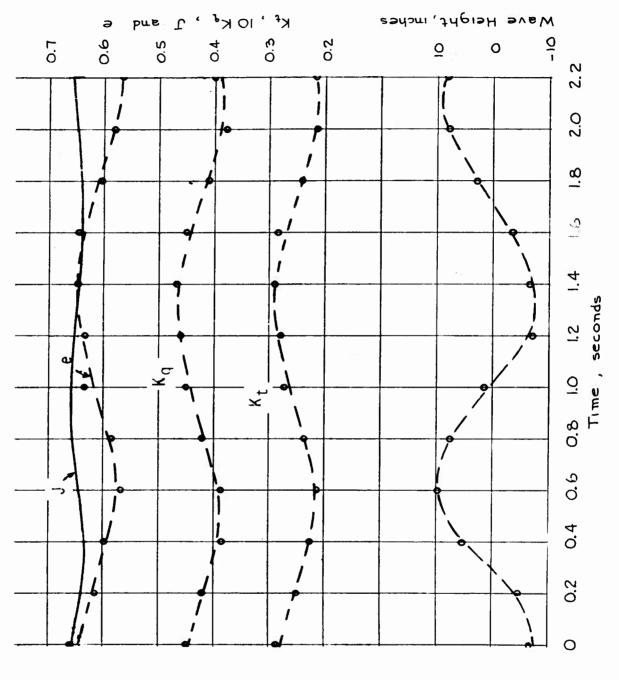
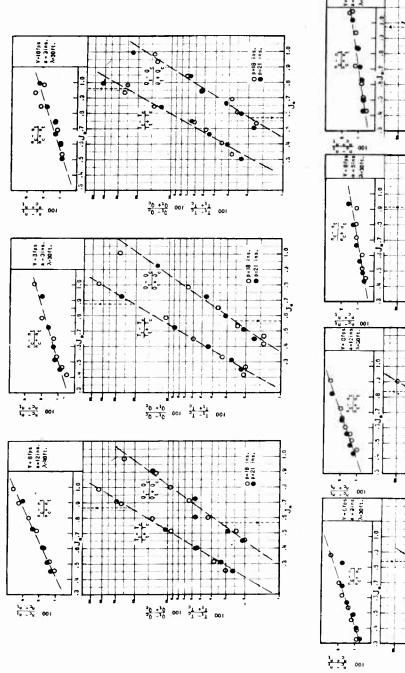
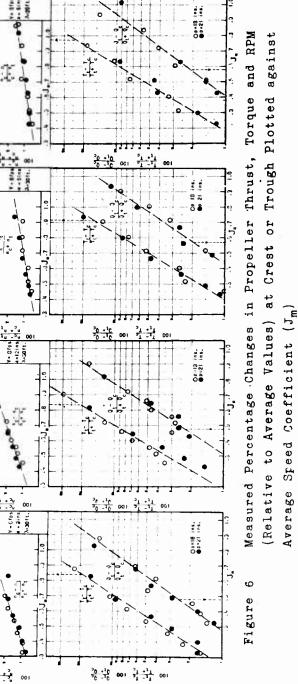
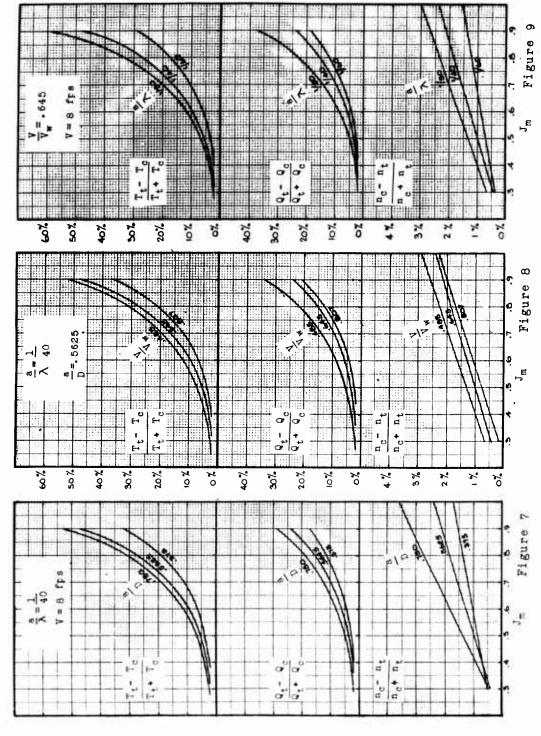


Figure 5 A Sample of Propeller Performance in Waves







Faired Curves of Percentage Changes in Propeller Thrust, Torque and RPM (Relative to Average Values) at Crest or Trough Plotted against Average Speed Coefficient $(\mathtt{J_m})$

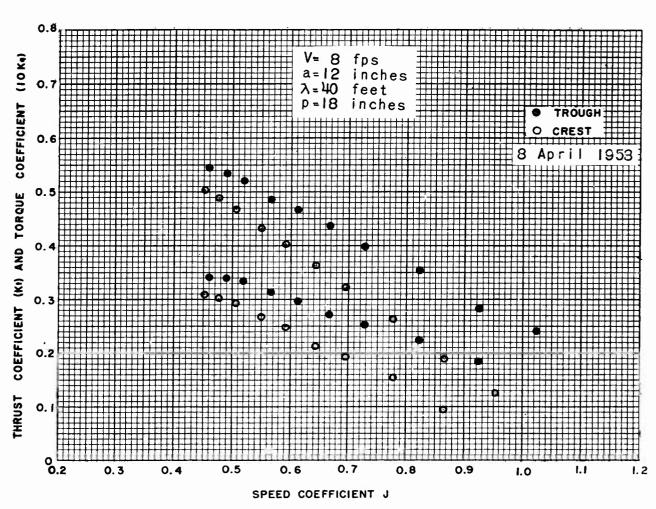


Figure 10 A Sample of Propeller Thrust and Torque Coefficients for Crest and Trough of a Wave, Plotted against Speed Coefficient (JI)

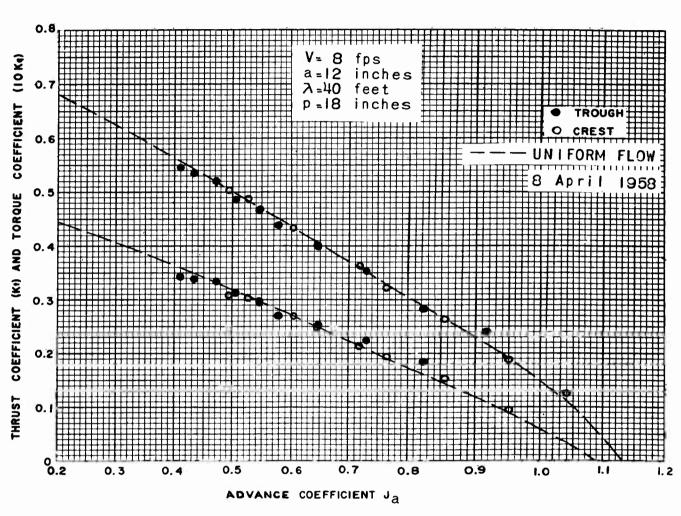
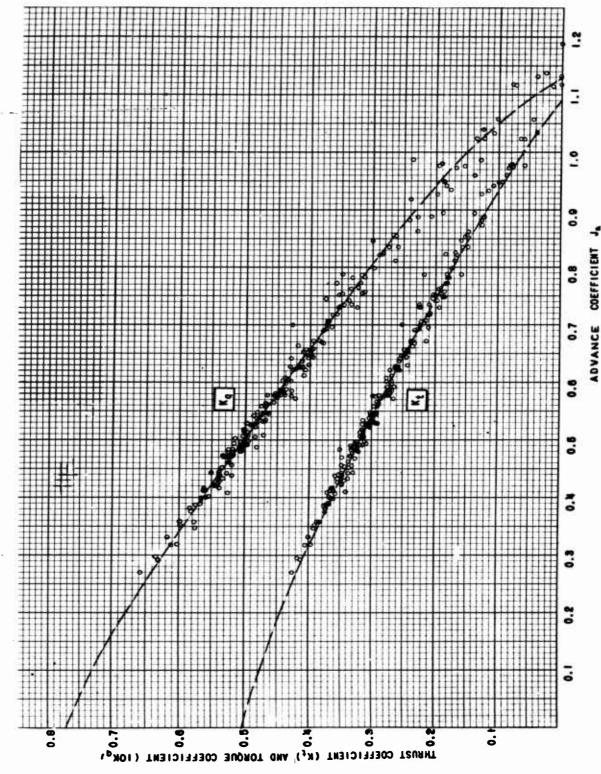


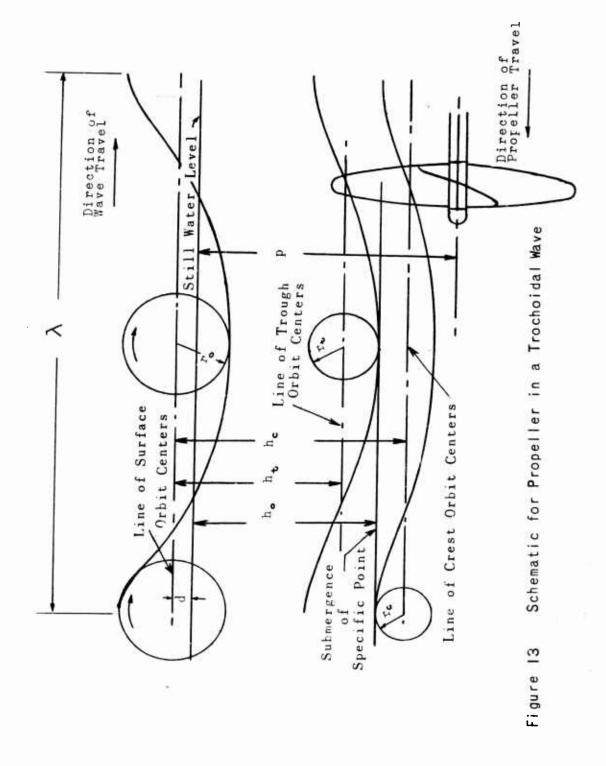
Figure II A Sample of Propeller Thrust and Torque Coefficients for Crest and Trough of a Wave, Plotted against Advance Coefficient (J_a)



Troughs of all Maves, Plotted against Advance Coefficient (Ja)

Propeller Thrust and Torque Coefficients for Crests and

Figure 12



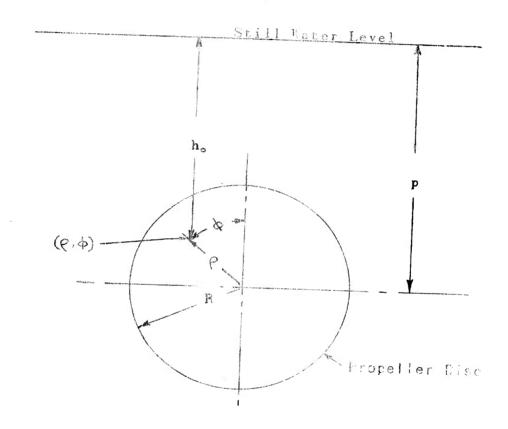


Figure 14 Coordinates for Propeller Disk

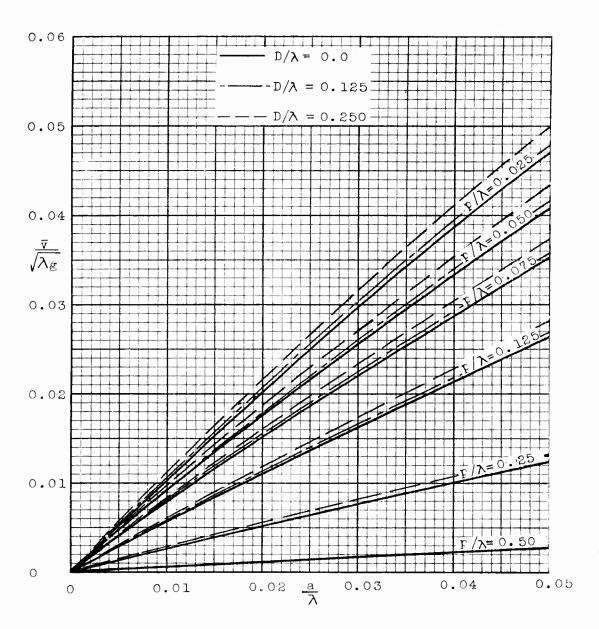


Figure 15 Mean Orbital Velocity below Crest of Trochoidal Wave.

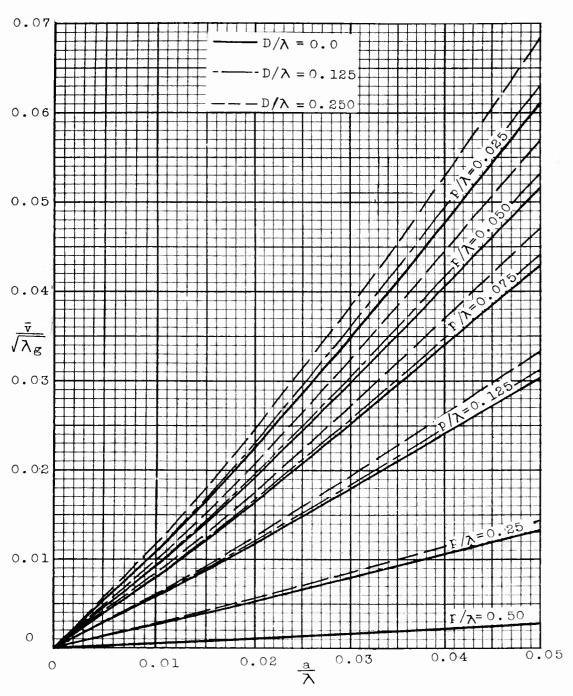


Figure 16 Mean Orbital Velocity below Trough of Trochoidal Wave.

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